Supporting Information


A Bidirectional Soft Diode for Artificial Systems

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This includes:

Supporting Text

Figs. S1 to S6

Captions for Movies S1 to S7

Other Supporting Information for this manuscript include the following:

Movies S1 to S7
Dimensional analysis of soft valve opening

Buckingham π theory was applied to data that characterized the opening pressure across a series of soft valves with varying geometries and material properties. We find that the pressure across the valve required to open the valve for flow, \( P_o \) [ML\(^{-1}\)T\(^{-2}\)], depends on the valve geometry (e.g., leaflet thickness, \( t \) [L], leaflet overlap length, \( \delta \) [L], and channel diameter, \( D \) [L]), materials properties of the valve (e.g., material elasticity, \( E \) [ML\(^{-1}\)T\(^{-2}\)]), fluid properties (e.g., density, \( \rho \) [ML\(^{-3}\)], and viscosity, \( \mu \) [ML\(^{-1}\)T\(^{-1}\)]), and flow conditions (e.g., flow rate, \( Q \) [L\(^3\)T\(^{-1}\)]):

\[
P_o = f(D, t, \delta, E, \rho, \mu).
\]

We choose the recurring set:

\[
D = [L] \rightarrow [L] = D
\]
\[
\rho = [ML^{-3}] \rightarrow [M] = \rho D^3
\]
\[
Q = [L^3T^{-1}] \rightarrow [T] = D^3/Q
\]

to develop five Π groups:

\[
P_o = \left[ \frac{M}{LT^2} \right] \rightarrow \Pi_1 = \frac{(P_o)(D)(D^3/Q)^2}{\rho D^3} = \frac{P_o D^4}{\rho Q^2}
\]
\[
t = [L] \rightarrow \Pi_1 = \frac{t}{D}
\]
\[
\delta = [L] \rightarrow \Pi_2 = \frac{\delta}{D}
\]
\[
E = \left[ \frac{M}{LT^2} \right] \rightarrow \Pi_3 = \frac{(E)(D)(D^3/Q)^2}{\rho D^3} = \frac{ED^4}{\rho Q^2}
\]
\[
\mu = \left[ \frac{M}{LT} \right] \rightarrow \Pi_4 = \frac{(\mu)(D)(D^3/Q)}{\rho D^3} = \frac{\mu D}{\rho Q}
\]

such that the following relationship collapses the experimental data into a single linear relationship:
$\frac{P_0 D^4}{\rho Q^2} = \left( \frac{t}{D} \right)^{1/2} \left( \frac{\delta}{D} \right)^{-1/2} \left( \frac{E D^4}{\rho Q^2} \right)^{1/2} \left( \frac{\mu D}{\rho Q} \right)^{1/2}$

Energy storage

The energy that can be stored in between a pair of soft diodes depends on the compressibility of the fluid medium ($\gamma$), expansivity of the storage cavity ($\Delta V_{\text{exp}}$), and the flipping characteristics of the diode (e.g., geometry and elasticity). Extractable energy can be calculated by tracking the work required to compress the fluid medium in the storage cavity and expand its elastic walls. This can be quantified as P-V work,

$$W = \int_{V_o}^{V_f} P \, dV,$$

where $dV$ includes the volume change in the storage cavity due to (1) fluid compressibility and (2) wall expansion. The amount of energy storage was measured using a pair of soft PDMS valves (Sylgard 184) (D = 6 mm, $t = 600 \, \mu m$, and $\delta = 2 \, mm$) with water as the liquid medium. This integral was evaluated by measuring the pressure change in the storage cavity as a known quantity of fluid was introduced ($\Delta V_t$) using a syringe pump. We measured an injection of 1 mL of water resulted in a storage capacity of 0.76 mJ using the pair of diodes (Figure S5).

Additional insight for the scalability of elastic energy storage can be described using a simple model. From the expression for P-V work, the amount of energy stored increases as the injected fluid volume increases. The injected fluid originates from volume changes due to the compressibility of the fluid ($\Delta V_c$) and the expansion of the storage cavity ($\Delta V_{\text{exp}}$), $\Delta V_t = \Delta V_c + \Delta V_{\text{exp}}$. This means strategies to increase either $\Delta V_c$ or $\Delta V_{\text{exp}}$ by using more compressible fluids (i.e., gases) or more expansive storage cavities are pathways to store more energy.

To show this, we consider storing energy with a liquid (i.e., water) that has an approximately constant compressibility ($\gamma = \text{con}$.) over the ranges of pressure considered ($P < \sim 2 \, \text{bar}$). Other working fluids, such as gases, can be interpreted by substituting an appropriate equation of state into the expression for P-V work. Given the low compressibility of water, we approximate the process of
energy storage by decoupling the effects of fluid compression and wall expansion, \( dV = dV_c + dV_{\text{exp}} \). This gives some insight into ways to increase the storage density without needing to acquire experimental data or integrate numerical simulations. If we further assume the expansion of the storage volume occurs at approximately uniform pressure, then \( W \) becomes,

\[
W = \int_{V_0}^{V_f} P \, dV_c + \Delta P \Delta V_{\text{exp}}.
\]

For water with \( \gamma = \text{con.} \), the definition of compressibility \( (\gamma = 1/\rho \frac{\partial \rho}{\partial P}) \) can be solved to yield \( \rho(p) = \rho_0(1 + \gamma P) \). We can use this process to approximate the first integral by assuming the applied force (i.e., injected pressure) acts as a piston to store energy in the compressible volume of the medium. If we assume this process is 1D (varies with \( h \)) with uniform area and mass is conserved in the compressible component of energy storage \( (\rho(P)/\rho_0 = V_0/V(P) = h_0/h(P)) \) then the pressure can be expressed as,

\[
p(V) = \frac{1}{\gamma} \left( \frac{h_0}{h(P)} - 1 \right),
\]

\[
W \approx \Delta P \left( \frac{1}{2} V_0 \Delta P + \Delta V_{\text{exp}} \right),
\]

where \( \frac{1}{2} V_0 \Delta P \) represents the volume change due to compressible effects in a fluid volume and \( \Delta P \) is the stored fluid pressure.

**Energy release**

The energy discharged by a soft PDMS valve (Sylgard 184) with valve diameter of 6 mm, leaflet thickness of 600 \( \mu \)m, and valve overlap of 2 mm was calculated by imaging the jet of water released from the valve (Movie S6). Here, the soft valve discharged energy at an average rate of \( \sim 0.35 \) W (Figure S5). The energy released is calculated by integrating the power, of the jet over the duration of release,

\[
W_{\text{released}} = \frac{1}{2} \rho A \int v^3 \, dt
\]
where the cross-sectional area of the jet, A, is taken to be equal to the flow area of the tubing (1/16” inner diameter), and the density of water was 1000 kg/m$^3$. The average velocity $v$ of the projectile jet was measured as a function of time by relating the kinematics of horizontal and vertical motion of the water jet (Figure S6). For a point along the water jet, the horizontal and vertical displacements of the point relative to the nozzle are $x(t) = v\tau$ and $y(t) = \frac{1}{2}g\tau^2$, respectively. We find that the velocity of the water jet exiting the nozzle is:

$$v(t) = \frac{x(t)}{\sqrt{\frac{2y(t)}{g}}}$$

Digital Logic in Soft Systems

Diodes are used to create elementary logic gates (AND, OR). Each gate functions using pressure as a logic variable with two inputs (A and B), a reference pressure ($P_{\text{ref}}$), and an output (Q). In each gate, pressure is treated as the logic variable and is assigned a 1 if $P - P_{\text{atm}} > P_0$ (i.e., fluid moves forward) and a 0 otherwise. In the OR gate, each diode is oriented in the same direction (leaflets facing the output), has its output (Q) tied together, and is connected to atmospheric pressure ($P_{\text{ref}} = P_{\text{atm}}$) through a pull-down resistor (Figure 4 of main text). This means if, for instance, A = 1 (diode input A has $P_A - P_{\text{atm}} > P_0$) and B = 0, the applied pressure will open diode A and flow will move to the output ($P_Q - P_{\text{atm}} > P_0$, $Q = 1$). Other combinations of inputs featuring a logic high input function similarly by enabling flow from the input to connect to the output by opening a soft diode.

Similarly, if A = 0 and B = 0, then both diodes remain closed and $P_Q - P_{\text{atm}} = 0$ which means $Q = 0$.

The AND gate features two distinct changes, (1) the diodes are flipped and face the inputs and (2) the common output is tied to a “high” reference pressure ($P_{\text{ref}} - P_{\text{atm}} > P_0$). The output pressure does not flip either diode due to the difference in flipping and opening pressures ($|P_{\text{in}} - P_{\text{ref}}| < P_f$). This means if, for instance, A = 1 (diode input A has $P_A - P_{\text{atm}} > P_0$) and B = 0 ($P_B = P_{\text{atm}}$), the reference pressure will open diode B and flow will move from the output to input B. Other combinations of inputs featuring a logic “low” value function similarly by forcing output flow to the gate and capping the pressure below the logic high threshold on the output. This means the output pressure will be $P_Q - P_0 < P_0$ and thus $P_Q - P_{\text{atm}} < P_0$ so $Q = 0$. Similarly, if A = 1 and B = 1, then both diodes remain closed and $P_Q - P_{\text{atm}} > P_0$ which means $Q = 1$. 


Analog Circuitry in Soft Systems

Soft diodes can also contribute to the control and processing of information in material systems. Each diode can rectify (i.e., block specific directions) and filter high-frequency signals (i.e., flows with short transients) depending on the state it is in (Figure 3B in main text). This could involve simply isolating signals from flowing in a particular direction or even eliminating sources of noise.

An advantage of these analog functions is that the computation occurs passively within the material system. There is no need for external power and no need for embedded electronics that complicate the system function and change the material properties. The diode is also able to be reconfigured to change its state and behavior while adding locally analog control in complex network assemblies.
Figure S1. Fabrication procedure used to manufacture soft diodes. Molds are filled with a desired elastomer (e.g., PDMS, Dragon Skin™ 10, etc.) and cured (step 1). Each mold encompasses half of the diode cross-section. Cured molds are extracted (step 2), painted with an elastomer (step 3), clamped, and cured to create a bonded diode (step 4). We note that the elastomer is only applied to edges of the channel and not the leaflets to ensure that the device is sealed but that flow is allowed through the valve.
**Figure S2.** Experimental setup used to measure fluid pressure throughout the study. A combination of a pressure transducer (Honeywell SSCDRNT1.6BAAA3) and microcontroller (Arduino Uno) were used to convert fluid pressure into a voltage and record its value over time. The pressure transducer was separated calibrated to correlate fluid pressure with output voltage.
**Figure S3.** Plot of the flipping pressure, $P_f$, over the course of 330 direction reversals. The valve had leaflet thickness $t \sim 600 \, \mu \text{m}$ and overlap $\delta \sim 2$ mm, channel diameter $D \sim 6$ mm, and was constructed of Sylgard 184 (PDMS). The pressure pulse used was generated by a constant flow rate of 1 mL/s.
Figure S4. Plot of the opening pressure, $P_o$, as a function of imposed volumetric flow rates, $Q$, for different leaflet geometries and valve materials. Each data point was averaged across three separate trials. Valve characteristics corresponding to leaflet thicknesses $t \sim 0.4$ mm to 0.8 mm, channel diameters $D \sim 0.4$ cm to 0.8 cm, leaflet overlaps $\delta \sim 1$ mm to 3 mm, and silicone elastomer materials including Sylgard 184 (1) and Dragonskin 10 (2) were examined.
Figure S5. A) Depiction of energy storage using soft diodes. Energy can be stored from (1) the expansion of elastic walls of the storage volume and (2) compressibility effects of the fluid medium. B) Measured P-V diagram of the charging process. Two PDMS diodes (Sylgard 184, D = 6 mm, t = 600 μm, and δ = 2 mm) stored ~ 0.76 mJ of energy after injecting ~ 1 mL of water.
Figure S6. Plot of the jet velocity resulting from the release of ~ 76 mJ of energy from a pair of cascaded soft diodes as a function of time. Each diode was molded using Sylgard 184 and featured a leaflet thickness of 0.6 mm, channel diameter of 6 mm, and leaflet overlap of 2 mm.
**Movie S1.**

Axial visualization of a diode opening and allowing fluid flow to pass through. The perspective shows deformation of the leaflets downstream of the applied pressure.

**Movie S2.**

Axial visualization of a diode opening and allowing fluid flow to pass through. The perspective shows deformation of the leaflets upstream of the applied pressure.

**Movie S3.**

Side-on visualization of a diode opening and allowing fluid flow to pass through.

**Movie S4.**

Visualization of the flipping (i.e., reconfiguring) process of a diode as it transitions from state 1 (right facing) to state 2 (left facing).

**Movie S5.**

Visualization of a pair of diodes configured in series to operate as a positive displacement pump analogous to the human heart.

**Movie S6.**

Visualization of a soft squid system being propelled in water by the release of energy from a pair of soft cascaded diodes.

**Movie S7.**

Visualization of a water jet caused by the release of energy from a pair of cascaded soft diodes ($W_{store}$ $\sim$ 76 mJ). The stream of water is initiated by the application of an external source of pressure (i.e., applied force) to actuate the storage volume, flip the orientation of a diode, and release the stored fluid pressure. The video was recorded with a fast-framing camera at 160 frames per second.